A New Theory of Gravity*

WEI-TOU NI

California Institute of Technology, Pasadena, California 91109

ABSTRACT

A new relativistic theory of gravity is presented. This theory agrees with all experiments to date. It is a metric theory; it is Lagrangian-based; and it possesses a preferred frame with conformally-flat space slices. With an appropriate choice of certain adjustable functions and parameters, this theory possesses precisely the same post-Newtonian limit as general relativity!

*Supported in part by the National Science Foundation [GP-27304, GP-28027] and the National Aeronautics and Space Administration [NGR 05-002-256].

11 p.

I. INTRODUCTION

Since 1970, the gravitation research group at Caltech has been analyzing the experimental foundations of relativistic theories of gravity. Our results to date are summarized in the "Varenna lecture notes" of Will.[1] Those results had led us to hope that current experiments were good enough to rule out all theories except (i) general relativity, and (ii) theories which reduce to general relativity when their adjustable parameters are appropriately adjusted (e.g., the Brans-Dicke-Jordan theory which reduces to general relativity as $\omega \to \infty$). We also had come to hope that general relativity could be distinguished from all other viable metric theories by the form of its post-Newtonian limit (PPN parameter values $\beta = \gamma = 1$, $\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$).

The purpose of this paper is to explode our hopes. More particularly, this paper will formulate a new theory of gravity which (for certain values of its adjustable parameters) has precisely the same post-Newtonian limit as general relativity, but can never reduce to general relativity in the full, non-linear case.

To distinguish experimentally between this new theory and general relativity, one will have to use non-post-Newtonian experiments. These could include: (i) gravitational-wave experiments, (ii) cosmological observations, and (iii) (in the distant future) post-post-Newtonian experiments. The present paper will not discuss such possibilities. Rather, it will merely present the new theory (§II) and compute its post-Newtonian limit (appendix).

II. PRESENTATION OF THE THEORY

We present the new theory using the notation and format of the author's recent "compendium of gravitation theories".[2] (In particular, note that we set $c = G = 1$.)

a. Gravitational fields present: A flat background metric $\boldsymbol{\eta} = \eta_{ij}\, \underset{\sim}{dx}^i \otimes \underset{\sim}{dx}^j$; scalar fields ϕ and t ; a one-form field $\underset{\sim}{\psi} = \psi_i\, \underset{\sim}{dx}^i$; and the physical metric $\mathbf{g} = g_{ij}\, \underset{\sim}{dx}^i \otimes \underset{\sim}{dx}^j$.

b. Arbitrary parameters and functions: Three arbitrary functions $f_1(\phi)$, $f_2(\phi)$, $f_3(\phi)$, and one arbitrary parameter e ; in the post-Newtonian limit, there are four arbitrary parameters, a, b, d, and e (see below).

c. Prior geometry: The following constraints are imposed, *à priori*, on the geometrical relationships among the gravitational fields:

(i) flatness of the metric $\boldsymbol{\eta}$

$$(\text{Riemann tensor constructed from } \boldsymbol{\eta}) = 0 \; ; \tag{1a}$$

(ii) "meshing constraints" on t and $\boldsymbol{\eta}$

$$t_{|ij} = 0 \quad , \tag{1b}$$

$$t_{,i}\, t_{,j}\, \eta^{ij} = +1 \quad , \tag{1c}$$

(Here and below a slash denotes a covariant derivative with respect to $\boldsymbol{\eta}$, and η^{ij} is the inverse of η_{ij}.)

$$t_{,i}\, \psi_j\, \eta^{ij} = 0 \; ; \tag{1d}$$

(iii) algebraic equation for the physical metric in terms of the "auxiliary gravitational fields" $\boldsymbol{\eta}$, ϕ , t , $\underset{\sim}{\psi}$

$$\mathbf{g} = f_2(\phi) \quad + [f_1(\phi) - f_2(\phi)]\underset{\sim}{dt} \otimes \underset{\sim}{dt} + \underset{\sim}{\psi} \otimes \underset{\sim}{dt}' + \underset{\sim}{dt} \otimes \underset{\sim}{\psi} \; . \tag{1e}$$

d. <u>Preferred coordinate system</u>: The prior-geometric constraints (1) guarantee the existence of a preferred coordinate system in which (i) the time coordinate is equal to the scalar field t ; (ii) the components of $\boldsymbol{\eta}$ are Minkowskiian

$$\eta_{ij} = \text{diagonal } (1,-1,-1,-1) \quad ; \tag{2a}$$

(iii) $\underset{\sim}{\psi}$ is purely spatial

$$\psi_o = 0 \quad ; \tag{2b}$$

(iv) the physical line element $g_{ij}dx^i dx^j$ is

$$ds^2 = f_1(\phi)dt^2 - f_2(\phi)(dx^2+dy^2+dz^2) + 2\psi_1 dxdt + 2\psi_2 dydt$$

$$+ 2\psi_1 dxdt + 2\psi_2 dydt + 2\psi_3 dzdt \quad . \tag{2c}$$

e. <u>Lagrangian</u>: The field equations are determined by an action principle

$$\delta \int \mathcal{L}\, d^4x = 0 \quad , \tag{3a}$$

where the Lagrangian density $\mathcal{L}$ is

$$\mathcal{L} = L_I\sqrt{-g} + \{(1/e)\psi_{i|k}\psi_{j|\ell}\eta^{ij}\eta^{k\ell} - 2\phi_{,i}\phi_{,j}\eta^{ij}$$

$$+ 2[f_3(\phi) + 1][\phi_{,i}t_{,j}\eta^{ij})^2\} \sqrt{-\eta} \tag{3b}$$

Here L_I is the standard "interaction Lagrangian" obtained by taking the standard Lagrangian for matter and nongravitational fields in flat spacetime, and replacing the Minkowskii metric by $\mathbf{g}$ (equivalence principle). The quantities $-g$ and $-\eta$ are the determinants of $||-g_{ij}||$ and $||-\eta_{ij}||$. In the action principle (3a) one is to vary the standard matter and nongravitational fields that appear in L_I, and the gravitational fields ϕ and $\underset{\sim}{\psi}$, while maintaining the prior-geometric constraints (1). In the preferred coordinate system (2) the Lagrangian density reduces to

$$\mathcal{L} = L_I\sqrt{-g} + (1/e)\psi_{\alpha,\beta}\,\psi_{\alpha,\beta} + 2\phi_{,\alpha}\,\phi_{,\alpha} + 2f_3(\phi)\phi_{,t}\,\phi_{,t} \tag{4}$$

(Summation on repeated Greek indices).

f. Field equations: The nongravitational field equations derived from this action principle take on their standard general relativistic form ("equivalence principle;" "comma-goes→to-semicolon rule"). In the preferred coordinate system, the gravitational field equations reduce to

$$\begin{aligned} &\psi_{\beta,\alpha\alpha} = 4\pi e\sqrt{-g}\;T^{0\beta} \\ &\phi_{,\alpha\alpha} + f_3(\phi)\phi_{,tt} + f_3'(\phi)\phi_{,t}\,\phi_{,t} - 2\pi\sqrt{-g}\;T^{ij}(\partial g_{ij}/\partial\phi) = 0 \end{aligned} \tag{5}$$

Here the stress-energy tensor is the same one as appears in the field equations of general relativity:

$$T_{ij} \equiv -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\;L_I)}{\partial g^{ij}} \quad ; \qquad T^{k\ell} \equiv g^{ik}g^{j\ell}T_{ij} \quad . \tag{6}$$

g. Post-Newtonian limit: Expand the arbitrary functions $f_1(\phi)$, $f_2(\phi)$, and $f_3(\phi)$ in powers of ϕ. In order that the metric will become flat in the absence of gravity ($\phi = \underset{\sim}{\psi} = 0$), require $f_1(0) = f_2(0) = 1$. In order that the theory will reduce to Newton's theory in the weak-field, slow-motion limit, require $f_1(\phi) = 1 - 2\phi + \cdots$. Define

a,b,d to be the coefficients of the first unconstrained terms ("post-Newtonian terms" in the expansions:

$$f_1(\phi) = 1 - 2\phi + 2b\phi^2 + \cdots , \tag{7a}$$

$$f_2(\phi) = 1 + 2a\phi + \cdots , \tag{7b}$$

$$f_3(\phi) = d + \cdots . \tag{7c}$$

Then the post-Newtonian limit of the theory reduces to the Nordtvedt-Will[3] PPN formalism with PPN parameter values

$$\gamma = a , \quad \beta = b , \quad \alpha_1 = -2e - 4a - 4 , \quad \alpha_2 = -d - 1 ,$$
$$\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 \quad . \tag{8}$$

(The proof is given in an appendix.)

h. Comparison with experiment. By comparing the PPN-parameter values (8) with the list of experimental limits on PPN parameters as given by Ni,[4] one learns that this theory agrees with all experiments to date if

$$\begin{aligned} 0.96 &< a < 1.12 \\ 0.84 &< b < 1.34 \\ -1.03 &< d < -0.97 \\ -2.2 &< e+2a < -1.8 \quad . \end{aligned} \tag{9}$$

i. Comparison with general relativity. Notice that if

$$a = b = 1 , \quad d = -1 , \quad e = -4 \tag{10}$$

then this theory has precisely the same post-Newtonian limit as general relativity! Thus, no post-Newtonian experiment can hope to make a clean distinction between this theory and general relativity.

j. Comparison with other Lagrangian-based theories. Will[5] and Ni[6] have shown that all Lagrangian-based metric theories must satisfy the PPN parameter constraints

$$\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 \quad . \tag{11}$$

Notice that the theory presented here has arbitrary values for all the remaining, unconstrained parameters. Thus, this theory possesses the most general post-Newtonian limit permitted for any Lagrangian-based metric theory.[7] This means that no post-Newtonian experiment can hope to make a clean distinction between this theory and any other Lagrangian based, metric theory.

k. Special cases. When the arbitrary functions $f_1(\phi)$, $f_2(\phi)$, and $f_3(\phi)$ are suitably specialized, one obtains the following theories: "Papapetrou I and II" [see §§III.D.v and vi of Reference 2], "Rosen" [see §III.D.ix of Reference 2], and Ni's "Lagrangian-based, stratified theory" [see §III.D.vii of Reference 2].

l. Conservation laws. Global conservation laws for this theory will be discussed in a future paper.

ACKNOWLEDGMENT

I am deeply obliged to Professor Kip S. Thorne for his great assistance, particularly with the presentation.

APPENDIX

COMPUTATION OF THE POST-NEWTONIAN LIMIT

To obtain the post-Newtonian limit of our theory, we proceed as follows. For convenience, we shall work in the preferred coordinate system. Let

$$\begin{aligned} \phi &= \phi_1 + \phi_2 + 0(6) \\ \psi_\beta &= \psi_{\beta 2} + 0(5) \end{aligned} \tag{A1}$$

where $\phi_1 = 0(2)$, $\phi_2 = 0(4)$, and $\psi_{\beta 2} = 0(3)$. [Here "$0(n)$" means of order "c^{-n}" in a post-Newtonian expansion.] Correct to post-Newtonian order, the field equations (5) are

$$\begin{aligned} -d\,\frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi_1 - \nabla^2\phi_2 &= 4\pi\rho[1 + (3a-1)U][1 + (2-2b)U \\ &\quad + v^2(1+a) + 3a\,\frac{p}{\rho} + \Pi] \;, \end{aligned} \tag{A2}$$

$$\nabla^2\psi_{\beta 2} = 4\pi e\;\rho\;v_\beta \;.$$

The $0(2)$ part of the field equations is

$$\nabla^2\phi_1 = -4\pi\rho \;, \tag{A3}$$

i.e.,

$$\phi_1 = U \tag{A4}$$

where U is the Newtonian potential. The $0(3)$ part is

$$\nabla^2\psi_{\beta 2} = 4\pi e\;\rho\;v_\beta \;, \tag{A5}$$

i.e.,

$$\psi_{\beta 2} = -eV_\beta = -e \int \frac{\rho(\underset{\sim}{x}',t)\ v_\beta(\underset{\sim}{x}',t)\,d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \tag{A6}$$

The $O(4)$ part is

$$\nabla^2\phi_2 = -dU_{,tt} - 4\pi\rho[(3a+1-2b)U + (1+a)v^2 + 3a\frac{p}{\rho} + \Pi] \tag{A7}$$

Let χ be the solution of

$$\nabla^2\chi = -2U \quad , \tag{A8}$$

i.e.,

$$\chi = -\int \rho|\underset{\sim}{x} - \underset{\sim}{x}'|d\underset{\sim}{x}' \quad . \tag{A9}$$

We can transform equation (A7) to

$$\nabla^2(\phi_2 - \frac{1}{2}d\cdot\chi_{,tt}) = -4\pi\rho[(3a+1-2b)U + (1+a)v^2 + 3a\frac{p}{\rho} + \Pi] \tag{A10}$$

Therefore

$$\phi_2 = \frac{1}{2}d\cdot\chi_{,tt} + 2\Phi \quad , \tag{A11}$$

where

$$\nabla^2\Phi = -4\pi\rho[\frac{1}{2}\Pi + \frac{1}{2}(3a+1-2b)U + \frac{1+a}{2}U + \frac{3}{2}a\frac{p}{\rho}] \quad . \tag{A12}$$

Combining equations (A4) and (A11), we find

$$\phi = U + 2\Phi + \frac{1}{2}d\chi_{,tt} + O(6) \quad . \tag{A13}$$

According to equations (2c), (A6) and (A13), the physical metric is

$$g_{oo} = 1 - 2U + 2bU^2 - 4\Phi - d\chi_{,tt} + O(6) \tag{A14}$$

$$g_{o\alpha} = -eV_\alpha \tag{A14}$$

$$g_{\alpha\beta} = -\delta_{\alpha\beta}(1+2aU) \quad .$$

By using the gauge transformation

$$\begin{aligned} x^{o\dagger} &= x^o + \frac{1}{2}\chi_{,o} \\ x^{\alpha\dagger} &= x^\alpha \quad , \end{aligned} \tag{A15}$$

we can transform the metric into the form

$$\begin{aligned} g_{oo}^{\dagger} &= 1 - 2U + 2bU^2 - 4\Phi + 0(6) \quad , \\ g_{o\alpha}^{\dagger} &= (\tfrac{1}{2}d - e)V_\alpha - \frac{1}{2}dW_\alpha + 0(5) \quad , \\ g_{\alpha\beta}^{\dagger} &= -(1 + 2aU) \quad , \end{aligned} \tag{A16}$$

where

$$W_\alpha(\underset{\sim}{x},t) = \int \frac{\rho(\underset{\sim}{x}',t)v_\beta(\underset{\sim}{x}')(x_\beta - x_\beta')(x_\alpha - x_\alpha')}{|\underset{\sim}{x} - \underset{\sim}{x}'|^3}\, dx' \tag{A17}$$

By comparing this with the PPN metric as given by Will and Nordtvedt,[8] we obtain the PPN parameter values listed in equation (8).

REFERENCES

1 C. M. Will, Lectures presented at the International School of Physics "Enrico Fermi", Varenna, Italy, July 17 to July 29, 1972 (to be published in the Proceedings of the School).

2 W.-T. Ni, Astrophys. J., in press.

3 C. M. Will and K. Nordtvedt, Jr., Astrophys. J., in press.

4 Reference 2.

5 C. M. Will, Astrophys. J. **169**, 125 (1971).

6 W.-T. Ni, paper in preparation.

7 Exception: One could conceive of--but one has no examples of--Lagrangian-based, metric theories with post-Newtonian limits that are more complex than the Nordvedt-Will 9-parameter formalism. Our results do not apply to such theories.

8 Reference 3.